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JHEP04(2009)028

(http://iopscience.iop.org/1126-6708/2009/04/028)

Published by IOP Publishing for SISSA
Received: *January 28, 2009*
Accepted: *March 5, 2009*
Published: *April 6, 2009*

JHEP04(2009)028

The non-anticommutative supersymmetric U_1 gauge theory

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Abstract: We discuss the non-anticommutative ($\mathcal{N} = \frac{1}{2}$) supersymmetric U_1 gauge theory in four dimensions, including a superpotential. We perform the one-loop renormalisation of the model, including the complete set of terms necessary for renormalisability, showing in detail how the eliminated and uneliminated forms of the theory lead to equivalent results.

Keywords: Renormalization Group, Supersymmetric gauge theory

ArXiv ePrint: 0901.2876

doi:10.1088/1126-6708/2009/04/028

Contents

1 Introduction

Deformed quantum field theories have been subject to renewed attention in recent years due to their natural appearance in string theory. Initial investigations focussed on theories on non-commutative spacetime in which the commutator of the spacetime co-ordinates becomes non-zero. More recently [1–9], non-anticommutative supersymmetric theories have been constructed by deforming the anticommutators of the Grassmann co-ordinates θ^{α} (while leaving the anticommutators of the $\overline{\theta}^{\dot{\alpha}}$ unaltered). Consequently, the anticommutators of the supersymmetry generators $\overline{Q}_{\dot{\alpha}}$ are deformed while those of the Q_{α} are unchanged. It is straightforward to construct non-anticommutative versions of ordinary supersymmetric theories by taking the superspace action and replacing ordinary products by the Moyal $*$-product [10] which implements the non-anticommutativity. Non-anticommutative versions of the Wess-Zumino model and supersymmetric gauge theories have been formulated in four dimensions [10, 11] and their renormalisability discussed [12–16], with explicit computations up to two loops [17] for the Wess-Zumino model and one loop for gauge theories [18–22]. Even more recently, non-anticommutative theories in two dimensions have been constructed [23, 25–28], and their one-loop divergences computed [24, 29]. In ref. [30] we returned to a closer examination of the non-anticommutative Wess-Zumino model (with a superpotential) in four dimensions, and showed that to correctly obtain results for the theory where the auxiliary fields have been eliminated, from the corresponding results for the uneliminated theory, it is necessary to include in the classical action separate couplings for all the terms which may be generated by the renormalisation process.

It seems natural to extend the above calculations to the gauged case, for which we seek the simplest possible gauged extension of the Wess-Zumino model with a (trilinear) superpotential. General gauged non-commutative theories were considered earlier [18–22], and in particular gauged interacting theories in ref. [22]; however there we only considered a trilinear superpotential in the adjoint SU_N case, and a mass term in the fundamental U_N case. The simplest model with a trilinear superpotential is the three-field Wess-Zumino model with a U_1 gauge invariance, and it is this model we shall consider here. We shall consider the one-loop renormalisation of this model in its entirety; the divergent contributions in the absence of a superpotential can be extracted from refs. [18, 19], while even some of the contributions with a superpotential may be extracted from ref. [22] by judicious adaptation of the results there presented for the case of the fundamental U_N case with mass terms; while a number of the divergent contributions will require a fresh diagrammatic computation. We start by considering the uneliminated theory and then proceed to compare with the results from the corresponding theory with the auxiliary fields eliminated.

2 Action

In this section we shall give the action for an $\mathcal{N} = \frac{1}{2}$ supersymmetric U_1 gauge theory coupled to chiral matter with a superpotential [10, 11, 22]. This is obtained by the reduction to components of the deformed, i.e. non-anticommutative, action in superspace. A U_1 gauge-invariant superpotential requires at least three chiral fields; we shall take exactly three, with scalar, fermion, auxiliary components denoted ϕ_i, ψ_i, F_i, $i = 1, 2, 3$. The corresponding U_1 charges are denoted q_i, $i = 1, 2, 3$. For simplicity we shall consider a massless superpotential. For convenience we split the action into kinetic and potential terms, namely

$$S_0 = S_{\text{kin}} + S_{\text{pot}} \tag{2.1}$$

where

$$\begin{aligned} S_{\text{kin}} = \int d^4x \Big[& -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - i\bar{\lambda}\overline{\sigma}^{\mu}(D_\mu\lambda) + \frac{1}{2}D^2 \\ & -igC^{\mu\nu}F_{\mu\nu}\bar{\lambda}\bar{\lambda} + \overline{F}_iF_i - i\overline{\psi}_i\overline{\sigma}^{\mu}(D_\mu\psi)_i - (D^\mu\overline{\phi})_i(D_\mu\phi)_i \\ & +\sqrt{2}gC^{\mu\nu}(D_\mu\overline{\phi})_i\bar{\lambda}\overline{\sigma}_\nu\psi_i + igC^{\mu\nu}\overline{\phi}_iF_{\mu\nu}F_i + \frac{1}{4}|C|^2g^2F_i\overline{\phi}_i\bar{\lambda}\bar{\lambda} \\ & +\sum_i \Big\{ gq_i\overline{\phi}_iD\phi_i + i\sqrt{2}gq_i(\overline{\phi}_i\lambda\psi_i - \overline{\psi}_i\bar{\lambda}\phi_i) \\ & -\gamma_iC^{\mu\nu}g\left[\sqrt{2}(D_\mu\overline{\phi})_i\bar{\lambda}\overline{\sigma}_\nu\psi_i + \sqrt{2}\overline{\phi}_i\bar{\lambda}\overline{\sigma}_\nu(D_\mu\psi)_i + i\overline{\phi}_iF_{\mu\nu}F_i\right]\Big\}\Big], \end{aligned} \tag{2.2}$$

and

$$\begin{aligned} S_{\text{pot}} = -\int d^4x \Big[& \{(F_iG_i - y\phi_1\psi_2\psi_3 - y\phi_2\psi_3\psi_1 - y\phi_3\psi_1\psi_2) + \text{h.c.}\} \\ & + 2ig\overline{y}C^{\mu\nu}F_{\mu\nu}\overline{\phi}_1\overline{\phi}_2\overline{\phi}_3 - \frac{1}{4}y|C|^2F_1F_2F_3\Big], \end{aligned} \tag{2.3}$$

where

$$G_1 = y\phi_2\phi_3, \tag{2.4}$$

and similarly for G_2, G_3 (corresponding to a superpotential $W(\Phi) = y\Phi_1\Phi_2\Phi_3$). The covariant derivative is defined by

$$(D_\mu\phi)_i = (\partial_\mu + igq_iA_\mu)\phi_i. \tag{2.5}$$

In eq. (2.2), $C^{\mu\nu}$ is related to the non-anti-commutativity parameter $C^{\alpha\beta}$ by

$$C^{\mu\nu} = C^{\alpha\beta}\epsilon_{\beta\gamma}\sigma^{\mu\nu\gamma}_{\alpha}, \tag{2.6}$$

where

$$\sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu\overline{\sigma}^\nu - \sigma^\nu\overline{\sigma}^\mu),$$
$$\overline{\sigma}^{\mu\nu} = \frac{1}{4}(\overline{\sigma}^\mu\sigma^\nu - \overline{\sigma}^\nu\sigma^\mu), \tag{2.7}$$

and

$$|C|^2 = C^{\mu\nu}C_{\mu\nu}. \tag{2.8}$$

Our conventions are in accord with ref. [10]; in particular,

$$\sigma^\mu\overline{\sigma}^\nu = -\eta^{\mu\nu} + 2\sigma^{\mu\nu}. \tag{2.9}$$

The definition of $|C|^2$ is similarly well-established although C^2 might be a preferable notation for this quantity.

For gauge invariance of S_{pot} we require

$$q_1 + q_2 + q_3 = 0, \tag{2.10}$$

while anomaly cancellation leads to

$$q_1q_2q_3 = 0 \tag{2.11}$$

so that the allowed set of charges is in fact $(q, -q, 0)$. This means that in fact the most general trilinear superpotential is in fact $W = y\Phi_1\Phi_2\Phi_3 + y'\Phi_3^3$ (assuming Φ_3 to be the neutral field). We choose, however, to retain $W = y\Phi_1\Phi_2\Phi_3$ and to present formulae in a manner explicitly symmetric under q_i permutations; for example for later convenience we denote

$$Q = q_1^2 + q_2^2 + q_3^2. \tag{2.12}$$

Note also that it follows from eqs. (2.10), (2.11) that superpotential mass terms are allowed in general; however as remarked earlier we will restrict ourselves to the massless case.

It is interesting to note that the constraints eqs. (2.10), (2.11) mean that if we set $q_1 = -q_2 = q$ and $y = \sqrt{2}gq$ then the undeformed theory has $\mathcal{N} = 2$ supersymmetry.

It is easy to show that S_0 is invariant under

$$\begin{aligned}
\delta A_\mu &= -i\bar{\lambda}\overline{\sigma}_\mu\epsilon,\\
\delta\lambda_\alpha &= i\epsilon_\alpha D + (\sigma^{\mu\nu}\epsilon)_\alpha\left[F_{\mu\nu} + \frac{1}{2}iC_{\mu\nu}\bar{\lambda}\bar{\lambda}\right], \quad \delta\bar{\lambda}_{\dot{\alpha}} = 0,\\
\delta D &= -\epsilon\sigma^\mu D_\mu\bar{\lambda},\\
\delta\phi_i &= \sqrt{2}\epsilon\psi_i, \quad \delta\overline{\phi}_i = 0,\\
\delta\psi_i^\alpha &= \sqrt{2}\epsilon^\alpha F_i, \quad \delta\overline{\psi}_{i\dot{\alpha}} = -i\sqrt{2}(D_\mu\overline{\phi}_i)(\epsilon\sigma^\mu)_{\dot{\alpha}},\\
\delta F_i &= 0, \quad \delta\overline{F}_i = -i\sqrt{2}D_\mu\overline{\psi}_i\overline{\sigma}^\mu\epsilon - 2igq_i\overline{\phi}_i\epsilon\lambda + 2C^{\mu\nu}gD_\mu(\overline{\phi}_i\epsilon\sigma_\nu\bar{\lambda}).
\end{aligned} \tag{2.13}$$

The set of terms multiplied by γ_i are separately $\mathcal{N} = \frac{1}{2}$ invariant under the transformations of eq. (2.13); they are not in fact produced by the reduction to components of the superspace action, but we have anticipated the need for them later when we renormalise the theory. It will be sufficient to take γ_i to consist purely of divergent contributions. The

$|C|^2F_1F_2F_3$ and $|C|^2F_i\overline{\phi}_i\bar{\lambda}\bar{\lambda}$ terms in eqs. (2.2), (2.3) are also each separately $\mathcal{N}=\frac{1}{2}$ invariant, and therefore could be omitted from our action without spoiling the $\mathcal{N}=\frac{1}{2}$ invariance. However, once we do include the $|C|^2F_1F_2F_3$ and $|C|^2F_i\overline{\phi}_i\bar{\lambda}\bar{\lambda}$ terms, it is necessary for the renormalisation of the model to include all possible terms which may be generated, as was explained in the ungauged case in ref. [30]. It is easy to list these terms [16, 22]. The action has a "pseudo R-symmetry" under

$$\phi_i \to e^{-i\alpha}\phi_i, \quad F_i \to e^{i\alpha}F_i, \quad \lambda \to e^{-i\alpha}\lambda, \quad C^{\alpha\beta} \to e^{-2i\alpha}C^{\alpha\beta}, \quad y \to e^{i\alpha}y, \tag{2.14}$$

$\overline{F}_i$, $\overline{\phi}_i$, $\bar{\lambda}$ and $\overline{y}$ transforming with opposite charges to F_i, ϕ_i, λ and y respectively, and all other fields being neutral; and also a "pseudo chiral symmetry" under

$$\phi_i \to e^{i\gamma}\phi_i, \quad y \to e^{-3i\gamma}y, \tag{2.15}$$

F_i and ψ_i transforming in a similar fashion to ϕ_i and barred quantities transforming with opposite charges; the gauge fields being unaffected. The divergent terms which can arise subject to these invariances, for the massless U_1 case and suppressing the $1,2,3$ subscripts, consist of (in addition to those already present in the action)

$$|C|^2F^2\overline{\phi}^2, \quad \overline{y}|C|^2F\overline{\phi}^4, \quad \overline{y}^2|C|^2\overline{\phi}^6, \quad \overline{y}|C|^2\bar{\lambda}\bar{\lambda}\overline{\phi}^3. \tag{2.16}$$

The combination

$$\overline{y}^{-1}[F_1\psi_2(C\psi_3) + F_2\psi_3(C\psi_1) + F_3\psi_1(C\psi_2)] \tag{2.17}$$

(where $(C\psi)_\alpha = C_{\alpha\beta}\psi^\beta$) is allowed by the above symmetries and $\mathcal{N}=\frac{1}{2}$ invariant, but we shall see later that it is not in fact generated as a divergence in the U_1 theory (at least at one loop) if it is not already present in the classical Lagrangian, and so we choose to omit it. Terms of the generic form $\overline{\phi}^2\psi(C\psi)$ are allowed by the above symmetries but it is impossible to construct an $\mathcal{N}=\frac{1}{2}$ invariant combination which includes them. We have included in (2.16) the appropriate factors of $\overline{y}$ for invariance under the pseudo-chiral symmetry. These factors are not uniquely determined since $y\overline{y}$ is invariant under this symmetry; the choice we have made is both concise and motivated by later considerations.

We must include all the terms in (2.16) with their own coefficient in the action and therefore we are led to our complete action

$$S = S_0 + S_{\text{gen}} \tag{2.18}$$

where S_0 is given in eq. (2.1) and

$$\begin{aligned} S_{\text{gen}} = \int d^4x \Big[& \overline{y}^{-1}|C|^2 \Big\{ \Big(k_1 - \frac{1}{4}y\overline{y}\Big) F_1F_2F_3 + k_2\left(F_1F_2\overline{G}_3 + F_2F_3\overline{G}_1 + F_3F_1\overline{G}_2\right) \\ & + k_3(F_1\overline{G}_2\overline{G}_3 + F_2\overline{G}_3\overline{G}_1 + F_3\overline{G}_1\overline{G}_2) + k_4\overline{G}_1\overline{G}_2\overline{G}_3 \Big\} \\ & + |C|^2 \Big\{ \Big(K_1 - \frac{1}{4}g^2\Big) F_i\overline{\phi}_i + K_2\overline{y}\overline{\phi}_1\overline{\phi}_2\overline{\phi}_3 \Big\} \bar{\lambda}\bar{\lambda} \Big]. \end{aligned} \tag{2.19}$$

(It is natural to impose the same cyclic symmetry on S_{gen} as already present in the superpotential). The $F_1F_2F_3$ and $F_i\overline{\phi}_i\bar{\lambda}\bar{\lambda}$ terms are now effectively assigned an arbitrary

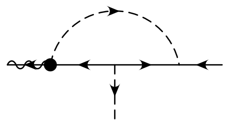

Figure 1. One-loop diagram with a C vertex and one gaugino, one ψ and one $\overline{\phi}$ external legs (a blob representing the C vertex and dashed, full, full/wavy lines representing scalar, fermion and gaugino fields respectively).

coefficient since the fact that they are separately $\mathcal{N}=\frac{1}{2}$ invariant (as are all the terms in S_{gen}) means there is no reason for their renormalisation to be accounted for purely by replacing quantities in S_0 by the corresponding bare ones; $\mathcal{N}=\frac{1}{2}$ invariance will not preserve the values of their coefficients derived from the deformed superfield action.

We use the standard gauge-fixing term

$$S_{\text{gf}}=\frac{1}{2\alpha}\int d^4x(\partial.A)^2 \tag{2.20}$$

with its associated ghost terms. The gauge propagator is given by

$$\Delta_{\mu\nu}=-\frac{1}{p^2}\left(\eta_{\mu\nu}+(\alpha-1)\frac{p_\mu p_\nu}{p^2}\right) \tag{2.21}$$

and the fermion propagator is

$$\Delta_{\alpha\dot{\alpha}}=\frac{p_\mu\sigma^\mu_{\alpha\dot{\alpha}}}{p^2}, \tag{2.22}$$

where the momentum enters at the end of the propagator with the undotted index.

3 Renormalisation

In this section we discuss the renormalisation of the gauged non-anticommutative Wess-Zumino model at one loop.

The divergent contributions from one-loop diagrams to terms in S_{kin} can mostly be extracted from the results for the $SU_N\times U_1$ case presented in refs. [18, 19], and so we shall just give the results (suppressing the well-known C-independent contributions) without tabulating the contributions from individual diagrams; an exception is the $y\overline{y}$-dependent divergences, since in ref. [22], where we incorporated a superpotential, we did not consider the resulting new divergent contributions to terms in S_{kin}. The corresponding diagrams are depicted in figures 1, 2. The contribution from figure 1 is simply

$$-2\sqrt{2}y\overline{y}gLC^{\mu\nu}\overline{\phi}_i\bar{\lambda}\overline{\sigma}_\nu\partial_\mu\psi_i, \tag{3.1}$$

where

$$L=\frac{1}{16\pi^2\epsilon}. \tag{3.2}$$

The contributions from figure 2 are tabulated in table 1, where

$$W_1=i\sqrt{2}y\overline{y}g^2C^{\mu\nu}A_\mu\sum_i q_i\overline{\phi}_i\bar{\lambda}\overline{\sigma}_\nu\psi_i. \tag{3.3}$$

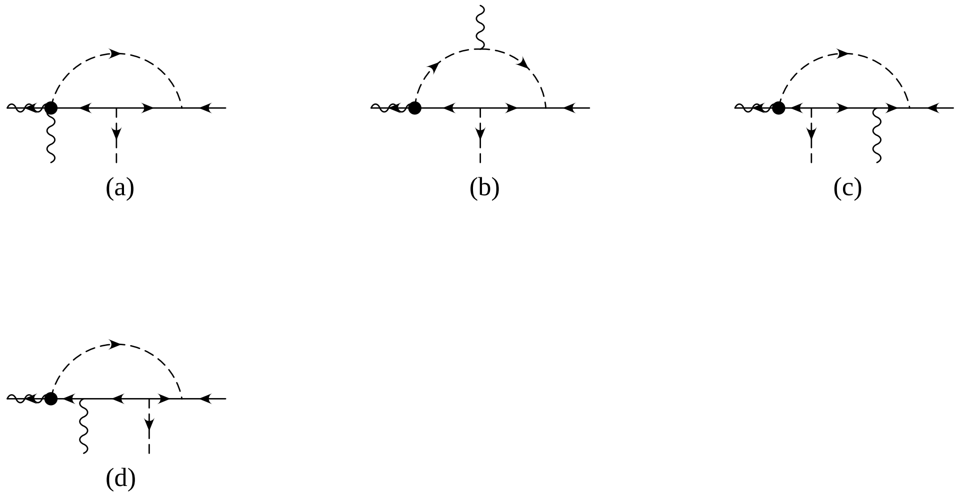

Figure 2. One-loop diagrams with a C vertex and one gauge, one gaugino, one ψ and one $\overline{\phi}$ external legs (wavy lines representing gauge fields).

a	$-2W_1$
b	W_1
c	$-W_1$
d	0

Table 1. Divergent contributions from figure 2.

(In this and all the following tables the factors of L are suppressed.) Taking into account the contributions from table 1, eq. (3.1) and those which can be extracted from ref. [19], we obtain

$$\Gamma_{\text{kin}}^{\text{pole}} = L\int d^4x\Big[-2ig^3QC^{\mu\nu}F_{\mu\nu}\bar{\lambda}\bar{\lambda} - 2\sqrt{2}gy\overline{y}C^{\mu\nu}\overline{\phi}_i\bar{\lambda}\bar{\sigma}_\nu D_\mu\psi_i \\ + \sum_i\Big(2\sqrt{2}\alpha g^3q_i^2C^{\mu\nu}D_\mu\overline{\phi}_i\bar{\lambda}\bar{\sigma}_\nu\psi_i - 2ig^3C^{\mu\nu}q_i^2\overline{\phi}_iF_{\mu\nu}F_i\Big)\Big]. \tag{3.4}$$

The contributions to S_{pot}, however, need to be reassessed due to the different form for the potential, and we therefore show the relevant diagrams in figure 3 and list the corresponding contributions in table 2. In table 2, W_2 and W_3 are defined by

$$\begin{aligned} W_2 &= iQg^3C^{\mu\nu}F_{\mu\nu}\overline{\phi}_1\overline{\phi}_2\overline{\phi}_3 \\ W_3 &= ig^3C^{\mu\nu}[q_1^2\partial_\mu\overline{\phi}_1\overline{\phi}_2\overline{\phi}_3 + q_2^2\partial_\mu\overline{\phi}_2\overline{\phi}_3\overline{\phi}_1 + q_3^2\partial_\mu\overline{\phi}_3\overline{\phi}_1\overline{\phi}_2]A_\nu. \end{aligned} \tag{3.5}$$

The contributions from table 2 add to

$$10iQg^3L\int d^4x\overline{y}C^{\mu\nu}F_{\mu\nu}\overline{\phi}_1\overline{\phi}_2\overline{\phi}_3. \tag{3.6}$$

Note that the contributions from figures 3(e)-(h) cancel those from 3(i)-(l); we shall subsequently omit several other pairs of diagrams where a similar cancellation occurs (in fact

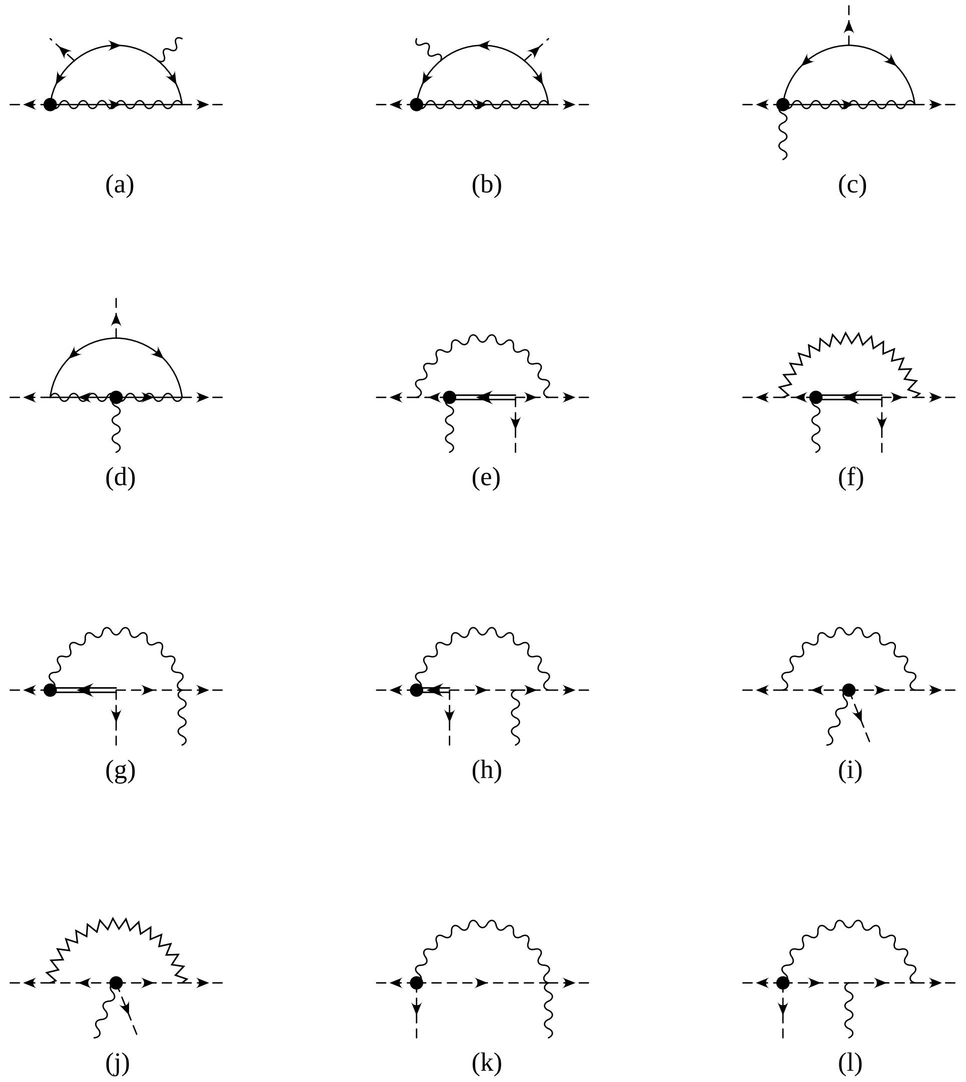

Figure 3. One-loop diagrams with a C vertex and three $\overline{\phi}$ and one gauge-field external legs (double, zigzag lines representing chiral and gauge auxiliary fields respectively).

we have done so already, since a potential divergent $y\overline{y}C^{\mu\nu}F_{\mu\nu}F\overline{\phi}$ contribution cancels for this reason).

The divergent contributions to the $F_1F_2F_3$ and $F_i\overline{\phi}_i\bar{\lambda}\bar{\lambda}$ terms will be given in detail shortly since these terms have now been assigned separate couplings in S_{gen} and so the divergences cannot be extracted from earlier work. The remaining divergent contributions are denoted by

$$\Gamma^{\text{pole}}_{\text{rem}} = -\int d^4x \Bigg[|C|^2\Bigg\{\overline{y}^{-1}[X_1F_1F_2F_3 + X_{2a}F_1F_2\overline{G}_3 + X_{2b}F_2F_3\overline{G}_1 + X_{2c}F_3F_1\overline{G}_2$$
$$+X_{3a}F_1\overline{G}_2\overline{G}_3 + X_{3b}F_2\overline{G}_3\overline{G}_1 + X_{3c}F_3\overline{G}_1\overline{G}_2 + X_4\overline{G}_1\overline{G}_2\overline{G}_3$$
$$+X_2'(F_1^2\overline{\phi}_1^2 + F_2^2\overline{\phi}_2^2 + F_3^2\overline{\phi}_3^2) + X_2''(q_1\overline{\phi}_1F_1 + q_2\overline{\phi}_2F_2 + q_3\overline{\phi}_3F_3)^2]$$

a	$4W_2 + 8W_3$
b	$4W_3$
c	$-2W_2 - 12W_3$
d	$8W_2$
e	$2\alpha W_2$
f	$2W_2$
g	$-4W_2 - 8W_3$
h	$8W_3$
i	$-2\alpha W_2$
j	$-2W_2$
k	$4W_2 + 8W_3$
l	$-8W_3$

Table 2. Divergent contributions from figure 3.

$$+ \left[X_5 F_i\overline{\phi}_i + X_5' \sum q_i^2 F_i\overline{\phi}_i + X_6\overline{y}\overline{\phi}_1\overline{\phi}_2\overline{\phi}_3\right]\bar{\lambda}\bar{\lambda}\Big\}$$
$$+X_7(q_1^2\overline{\phi}_1\psi_1 + q_2^2\overline{\phi}_2\psi_2 + q_3^2\overline{\phi}_3\psi_3)(q_1\overline{\phi}_1 C\psi_1 + q_2\overline{\phi}_2 C\psi_2 + q_3\overline{\phi}_3 C\psi_3)\Big]. \qquad (3.7)$$

(Note the overall minus sign, introduced to avoid a proliferation of negative signs later on.) In figures 4–9 are depicted the divergent one-loop diagrams contributing to X_1, etc. Their divergent contributions are shown diagram by diagram in tables 3–9 and given in total by

$$\begin{aligned}
X_1^{(1)} &= (6k_2 - 6g^2)y\overline{y}L,\\
X_{2a}^{(1)} &= \{4(k_1 + 2k_2 + 2k_3)y\overline{y} + 2(1+\alpha)k_2q_1q_2g^2\}L,\\
X_{2b}^{(1)} &= \{4(k_1 + 2k_2 + 2k_3)y\overline{y} + 2(1+\alpha)k_2q_2q_3g^2\}L,\\
X_{2c}^{(1)} &= \{4(k_1 + 2k_2 + 2k_3)y\overline{y} + 2(1+\alpha)k_2q_3q_1g^2\}L,\\
X_{3a}^{(1)} &= \{2(3k_2 + 6k_3 + 4k_4)y\overline{y} + (1+\alpha)[2(k_1 + 2k_2)q_2q_3 - Qk_3]g^2\}L,\\
X_{3b}^{(1)} &= \{2(3k_2 + 6k_3 + 4k_4)y\overline{y} + (1+\alpha)[2(k_1 + 2k_2)q_3q_1 - Qk_3]g^2\}L,\\
X_{3c}^{(1)} &= \{2(3k_2 + 6k_3 + 4k_4)y\overline{y} + (1+\alpha)[2(k_1 + 2k_2)q_1q_2 - Qk_3]g^2\}L,\\
X_4^{(1)} &= -(1+\alpha)(k_2 + 2k_3 + 2k_4)Qg^2L,\\
X_2'^{(1)} &= 2\,(k_1 + 2k_2 + k_3)\,y\overline{y}L,\\
X_2''^{(1)} &= -\frac{1}{4}(1+\alpha)g^4\overline{y},\\
X_5^{(1)} &= [(4K_1 + 2K_2)y\overline{y} - g^2y\overline{y})L,\\
X_{5'}^{(1)} &= g^2(8K_1 - 10g^2)L,\\
X_6^{(1)} &= [2(7-\alpha)K_1 + (7-\alpha)K_2 + 14g^2]Qg^2L,\\
X_7^{(1)} &= 16g^4L.
\end{aligned} \qquad (3.8)$$

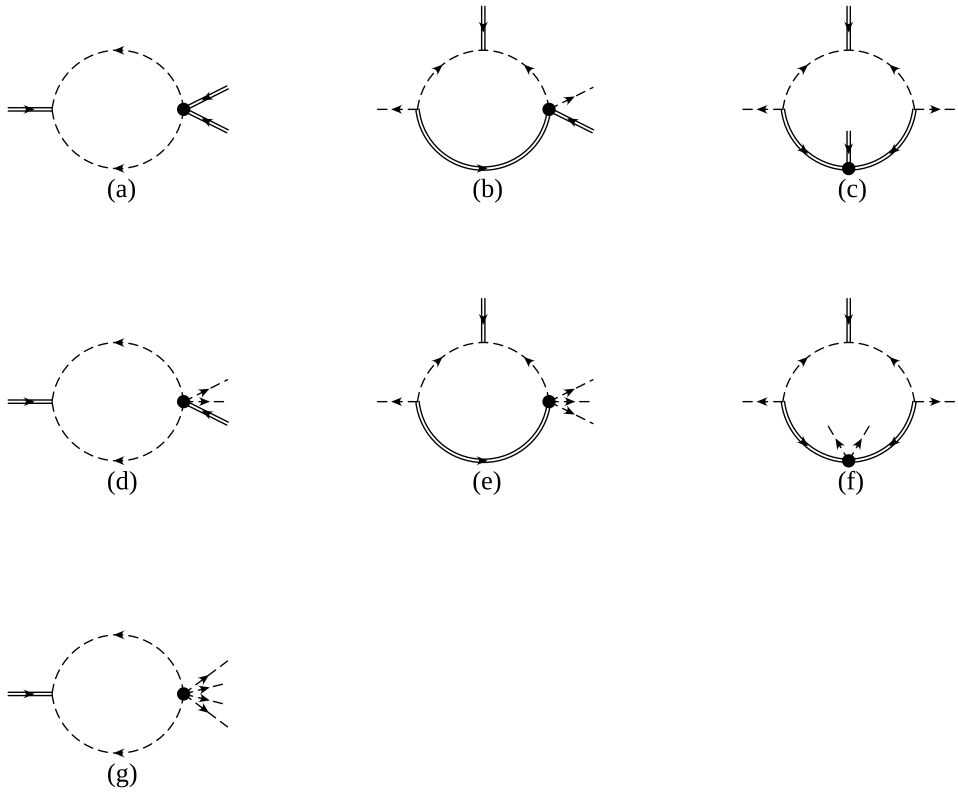

Figure 4. One-loop diagrams with a $|C|^2$ vertex, F or $\overline{\phi}$ external legs and purely F or $\overline{\phi}$ internal propagators.

	X_1	$X_{2a,b,c}$	$X_{3a,b,c}$	X'_2
a	$6k_2y\overline{y}$			
b		$8k_2y\overline{y}$		$4k_2y\overline{y}$
c		$4k_1y\overline{y}$		$2k_1y\overline{y}$
d		$8k_3y\overline{y}$		$2k_3y\overline{y}$
e			$12k_3y\overline{y}$	
f			$6k_2y\overline{y}$	
g			$8k_4y\overline{y}$	

Table 3. Divergent contributions from figure 4.

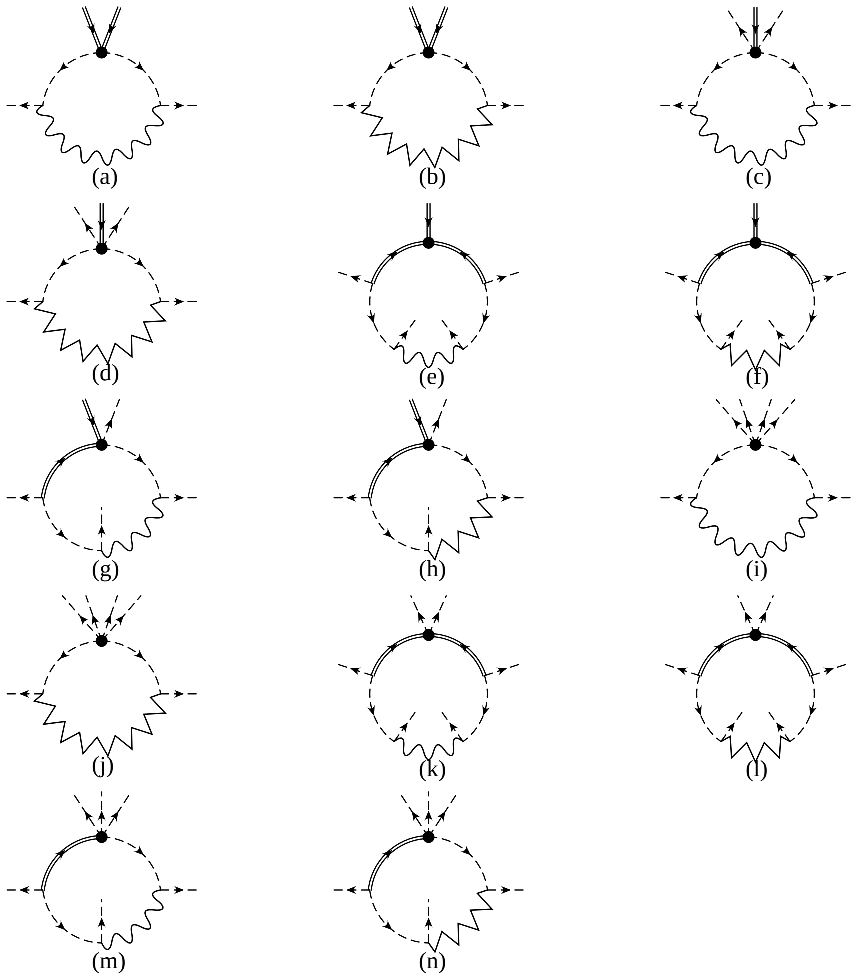

Figure 5. One-loop diagrams with a $|C|^2$ vertex, F or $\overline{\phi}$ external legs and an internal gauge or D propagator.

	X_{2a}	X_{2b}	X_{2c}	X_{3a}	X_{3b}	X_{3c}	X_4
a	$2\alpha k_2 q_1 q_2 g^2$	$2\alpha k_2 q_2 q_3 g^2$	$2\alpha k_2 q_3 q_1 g^2$				
b	$2k_2 q_1 q_2 g^2$	$2k_2 q_2 q_3 g^2$	$2k_2 q_3 q_1 g^2$				
c				$-\alpha k_3 Q g^2$	$-\alpha k_3 Q g^2$	$-\alpha k_3 Q g^2$	
d				$-k_3 Q g^2$	$-k_3 Q g^2$	$-k_3 Q g^2$	
e				$2\alpha k_1 q_2 q_3 g^2$	$2\alpha k_1 q_3 q_1 g^2$	$2\alpha k_1 q_1 q_2 g^2$	
f				$2k_1 q_2 q_3 g^2$	$2k_1 q_3 q_1 g^2$	$2k_1 q_1 q_2 g^2$	
g				$4\alpha k_2 q_2 q_3 g^2$	$4\alpha k_2 q_3 q_1 g^2$	$4\alpha k_2 q_1 q_2 g^2$	
h				$4k_2 q_2 q_3 g^2$	$4k_2 q_3 q_1 g^2$	$4k_2 q_1 q_2 g^2$	
i							$-2\alpha k_4 Q g^2$
j							$-2k_4 Q g^2$
k							$-\alpha k_2 g^2 Q$
l							$-k_2 Q g^2$
m							$-2\alpha k_3 Q g^2$
n							$-2k_3 Q g^2$

Table 4. Divergent contributions from figure 5.

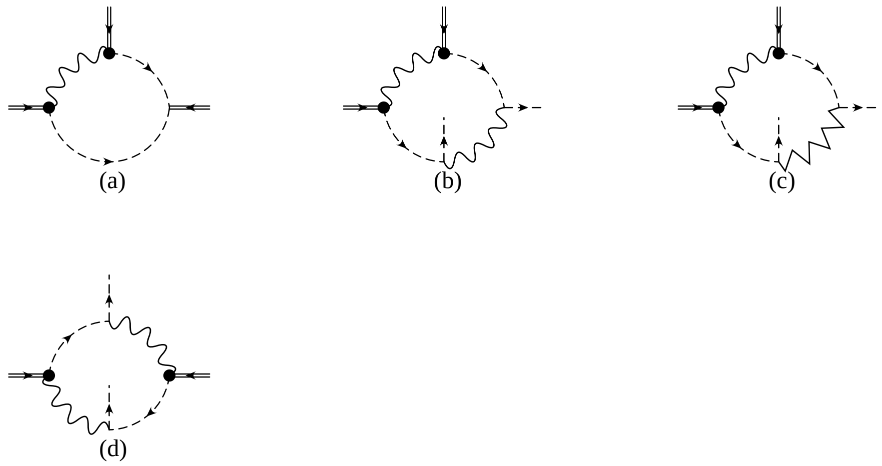

Figure 6. One-loop diagrams with two $C^{\mu\nu}$ vertices, F or $\overline{\phi}$ external legs and an internal gauge or D propagator.

	X_1	X_2''
a	$-6y\overline{y}g^2$	
b		$-\frac{1}{4}\alpha g^4$
c		$-\frac{1}{4}g^4$
d		0

Table 5. Divergent contributions from figure 6.

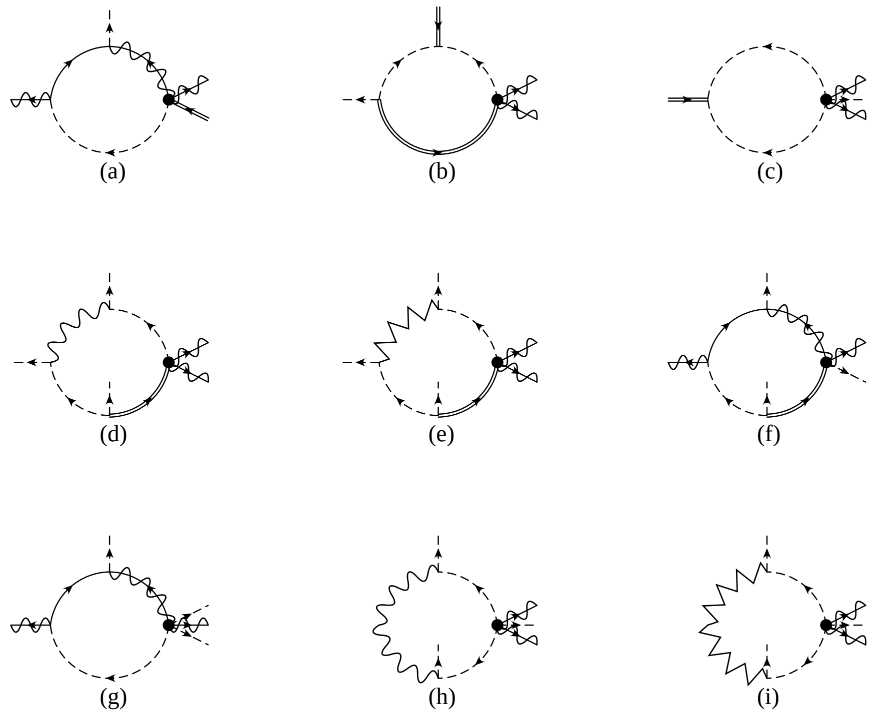

Figure 7. One-loop diagrams with a $|C|^2$ vertex, and two gaugino and F or $\overline{\phi}$ external legs.

	X_5	X_6	X'_5
a			$8g^2K_1$
b	$4K_1y\overline{y}$		
c	$2K_2y\overline{y}$		
d		$-2\alpha Qg^2K_1$	
e		$-2g^2QK_1$	
f		$16Qg^2K_1$	
g		$8Qg^2K_2$	
h		$-\alpha Qg^2K_2$	
i		$-Qg^2K_2$	

Table 6. Divergent contributions from figure 7.

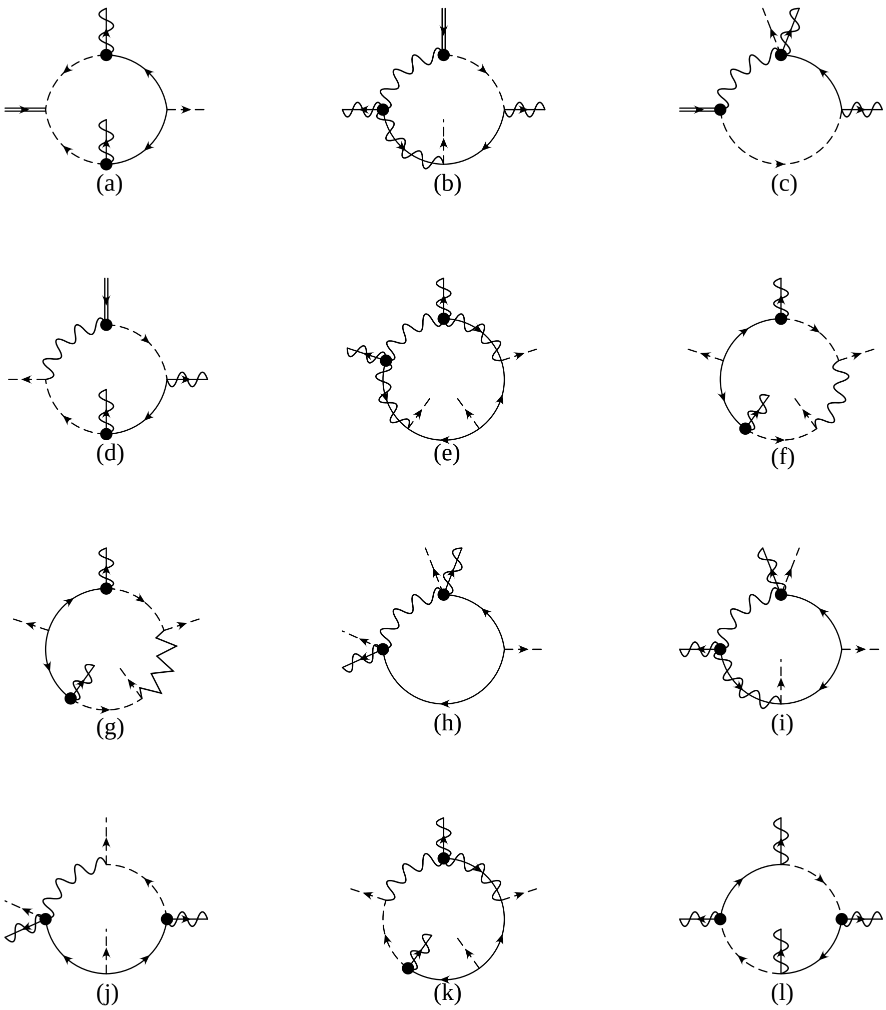

Figure 8. One-loop diagrams with two $C^{\mu\nu}$ vertices, and two gaugino and F or $\overline{\phi}$ external legs.

JHEP04(2009)028

	X_5	X_6	X_5'
a	$-g^2y\overline{y}$		
b			$-8g^4$
c			$-2g^4$
d			0
e		$8g^4Q$	
f		$\frac{1}{2}\alpha Qg^4$	
g		$\frac{1}{2}Qg^4$	
h		$\frac{1}{2}(3+\alpha)Qg^4$	
i		$4Qg^4$	
j		$-\alpha Qg^4$	
k		0	

Table 7. Divergent contributions from figure 8.

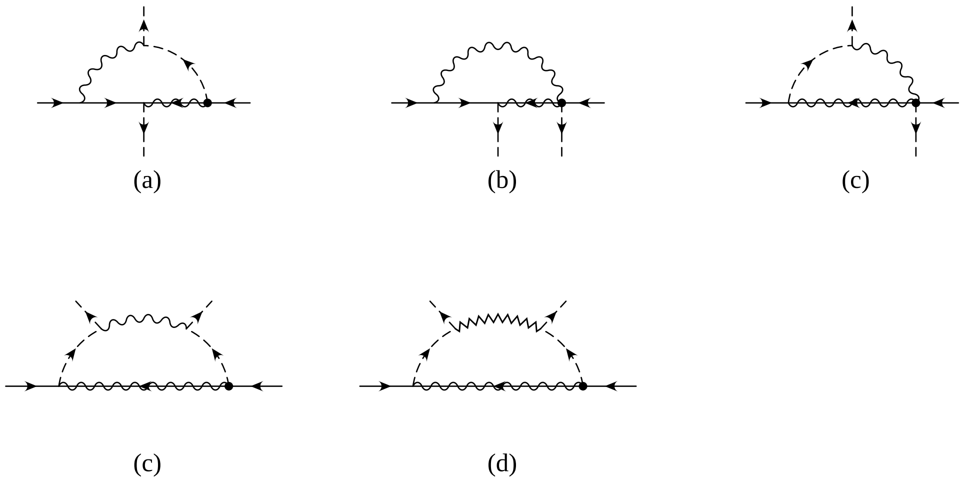

Figure 9. One-loop diagrams with two $\overline{\phi}$ and two ψ external legs (and no Yukawa vertices).

	X_7
a	$-4\alpha g^4$
b	$4(3+\alpha)g^4$
c	$-4\alpha g^4$
d	$4\alpha g^4$
e	$4g^4$

Table 8. Divergent contributions from figure 9.

The terms involving X_2', X_2'' and X_5' are not contained in the original action; while the term involving X_7 is not $\mathcal{N}=\frac{1}{2}$ invariant. However, we shall see later that all these terms may be removed (at least at one loop) by field redefinitions. Other diagrams which

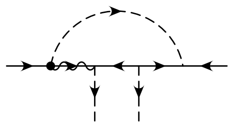

Figure 10. One-loop diagram with two $\overline{\phi}$ and two ψ external legs (and two Yukawa vertices).

Figure 11. One-loop diagrams with one F and two ψ external legs.

potentially contribute divergences turn out to be zero or to cancel. Figure 10 is in fact zero by symmetry. The divergences from the diagrams of figure 11 are of the form

$$\overline{y}^{-1}[(q_2-q_3)F_1\psi_2(C\psi_3)+(q_3-q_1)F_2\psi_3(C\psi_1)+(q_1-q_2)F_3\psi_1(C\psi_2)] \qquad (3.9)$$

which (in contrast to the similar combination in (2.17)) is also not $\mathcal{N}=\frac{1}{2}$ invariant; moreover there is no field redefinition which could remove these terms and so they must and indeed do cancel.

The divergences in eq. (3.8) should be cancelled as usual by replacing the parameters y, $\overline{y}$, g, k_{1-4}, $K_{1,2}$ and the fields ϕ_i, $\overline{\phi}_i$, F_i, $\overline{F}_i$, ψ_i, $\overline{\psi}_i$, λ, $\bar{\lambda}$ by corresponding appropriately-chosen bare quantities y_B, $\overline{y}_B$, k_{1B-4B}, $K_{1B,2B}$, ϕ_{iB}, $\overline{\phi}_{iB}$, F_{iB}, $\overline{F}_{iB}$, ψ_{iB}, $\overline{\psi}_{iB}$, λ_B, $\bar{\lambda}_B$, with the bare fields given by $\phi_{iB}=Z_{\phi_i}^{\frac{1}{2}}\phi_i$, etc. However, as emphasised in ref. [31], renormalisation of a gauged supersymmetric theory in the uneliminated case (i.e. without eliminating the auxiliary fields F_i and D) requires in general a non-linear renormalisation of F_i and D; and in the general $\mathcal{N}=\frac{1}{2}$ case in ref. [22] we also required a non-linear renormalisation of the gaugino field. In our present case we find it necessary to take at one loop

$$\begin{aligned}
F_{1B}^{(1)} &= Z_F^{\frac{1}{2}(1)}F_1-(\alpha+3)q_1^2g^2\overline{y}L\overline{\phi}_2\overline{\phi}_3,\\
\overline{F}_{1B}^{(1)} &= Z_F^{\frac{1}{2}(1)}F_1-(\alpha+3)q_1^2g^2yL\phi_2\phi_3+(\alpha+9)ig^2q_1^2gLC^{\mu\nu}F_{\mu\nu}\overline{\phi}_1\\
&\quad +k_1g^2L\left[\frac{1}{2}(\alpha+3)(q_3^2F_2\overline{\phi}_1\overline{\phi}_2+q_2^2F_3\overline{\phi}_1\overline{\phi}_3)+\alpha\overline{y}\left(q_1^2-\frac{1}{2}q_2^2-\frac{1}{2}q_3^2\right)\overline{\phi}_1^2\overline{\phi}_2\overline{\phi}_3\right.\\
&\qquad\qquad \left.+\,\overline{y}\left(q_1^2+\frac{1}{2}q_2^2+\frac{1}{2}q_3^2\right)\overline{\phi}_1^2\overline{\phi}_2\overline{\phi}_3\right]\\
&\quad +k_2g^2L\left[\frac{1}{2}\alpha(q_3^2F_2\overline{\phi}_1\overline{\phi}_2+q_2^2F_3\overline{\phi}_1\overline{\phi}_3)+\alpha\overline{y}\left(2q_1^2-\frac{1}{2}q_2^2-\frac{1}{2}q_3^2\right)\overline{\phi}_1^2\overline{\phi}_2\overline{\phi}_3\right.\\
&\qquad\qquad \left.-\left(q_1^2+q_2^2-\frac{1}{2}q_3^2\right)F_2\overline{\phi}_1\overline{\phi}_2-\left(q_1^2+q_3^2-\frac{1}{2}q_2^2\right)F_3\overline{\phi}_1\overline{\phi}_3+\frac{1}{2}\overline{y}(q_2^2+q_3^2)\overline{\phi}_1^2\overline{\phi}_2\overline{\phi}_3\right]\\
&\quad +k_3g^2\overline{y}L\left[\alpha q_1^2-(3q_1^2+2q_2^2+2q_3^2)\right]\overline{\phi}_1^2\overline{\phi}_2\overline{\phi}_3+2(k_1+2k_2+k_3)y\overline{y}LF_1\overline{\phi}_1^2
\end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{4}(1+\alpha)g^4 q_1\overline{\phi}_1(q_1F_1\overline{\phi}_1+q_2F_2\overline{\phi}_2+q_3F_3\overline{\phi}_3)+[-10g^2+(7+\alpha)K_1]g^2Lq_i^2\overline{\phi}_i\bar{\lambda}\bar{\lambda}\\
&-\frac{1}{3}Qg^2L\left[2\overline{y}^{-1}k_1F_2F_3+k_1(F_2\overline{\phi}_1\overline{\phi}_2+F_3\overline{\phi}_1\overline{\phi}_3)+(2k_1-6k_3)\overline{y}\overline{\phi}_1^2\overline{\phi}_2\overline{\phi}_3\right]\\
&+\overline{y}^{-1}\left[R^{(1)}F_2F_3+S^{(1)}(F_2\overline{G}_3+F_3\overline{G}_2)+T^{(1)}\overline{G}_2\overline{G}_3\right] \qquad (3.10)
\end{aligned}
$$

with similar expressions for $F^{(1)}_{2B,3B}$, and also

$$\lambda_B^{(1)} = Z_\lambda^{\frac{1}{2}(1)}\lambda + i\sqrt{2}g\sum_i \rho_i^{(1)}\overline{\phi}_i(C\psi_i). \qquad (3.11)$$

Here, Z_F and Z_λ, together with the renormalisation constants for the other fields have a loop expansion

$$Z_F = 1+\sum_{n\geq 1} Z_F^{(n)}, \qquad (3.12)$$

etc, and at one loop we have

$$
\begin{aligned}
Z_\lambda^{(1)} &= -2g^2LQ,\\
Z_A^{(1)} &= -2g^2LQ,\\
Z_g^{(1)} &= g^2LQ,\\
Z_F^{(1)} &= -2Ly\overline{y},\\
Z_{\phi_i}^{(1)} &= 2L\left[-y\overline{y}+(1-\alpha)g^2q_i^2\right], \quad i=1,2,3\\
Z_{\psi_i}^{(1)} &= 2L\left[-y\overline{y}-(1+\alpha)g^2q_i^2\right], \quad i=1,2,3. \qquad (3.13)
\end{aligned}
$$

The presence of ρ_i in the bare action produces terms

$$\sum_i \rho_i g\left[\sqrt{2}C^{\mu\nu}(D_\mu\overline{\phi}_i\bar{\lambda}\overline{\sigma}_\nu\psi_i+\overline{\phi}_i\bar{\lambda}\overline{\sigma}_\nu D_\mu\psi_i)+2\overline{\phi}_i\psi_i(\sum q_j\overline{\phi}_jC\psi_j)\right]. \qquad (3.14)$$

The ρ_i in eq. (3.11) are, like the γ_i in eq. (2.2), purely divergent quantities, and at one loop we find we need to take

$$
\begin{aligned}
\gamma_i^{(1)} &= (8g^2q_i^2-2y\overline{y})L,\\
\rho_i^{(1)} &= 8g^2q_i^2L. \qquad (3.15)
\end{aligned}
$$

With this value for ρ_i, the $\mathcal{N}=\frac{1}{2}$ non-invariant terms involving X_7 in eq. (3.7) are cancelled at one loop. In eq. (3.10), R, S, T represent possible additional renormalisations of F_i which are not determined by the requirements of renormalisability.

With the above expression for $F_{iB}^{(1)}$, the renormalisation of the Yukawa couplings is as expected from applying the non-renormalisation theorem in the superfield context, namely

$$y_B=\mu^{\frac{1}{2}\epsilon}Z_{\Phi_1}^{-\frac{1}{2}}Z_{\Phi_2}^{-\frac{1}{2}}Z_{\Phi_3}^{-\frac{1}{2}}y, \quad \overline{y}_B=\mu^{\frac{1}{2}\epsilon}Z_{\Phi_1}^{-\frac{1}{2}}Z_{\Phi_2}^{-\frac{1}{2}}Z_{\Phi_3}^{-\frac{1}{2}}\overline{y}, \qquad (3.16)$$

where μ is the usual dimensional regularisation mass parameter, and Z_{Φ_i}, $i=1,2,3$ are the renormalisation constants for the chiral superfields as computed in a supersymmetric gauge, namely (at one loop)

$$Z_{\Phi_i}^{(1)} = 2L\left[-y\overline{y}+2g^2q_i^2\right], \quad i=1,2,3. \qquad (3.17)$$

The β-function for y is defined by $\beta_y = \mu\frac{d}{d\mu}y$ with a similar expression for $\beta_{\overline{y}}$ and then by virtue of eqs. (3.16), (3.17),

$$\beta_y^{(1)} = \frac{1}{16\pi^2}(3y\overline{y} - 2g^2 Q)y, \tag{3.18}$$

with a similar expression for $\beta_{\overline{y}}^{(1)}$.

Note that if we set $q_1 = -q_2 = q$ and $y = \overline{y} = \sqrt{2}gq$ then eq. (3.19) reduces to

$$\beta_g^{(1)} = 2q^2\frac{g^3}{16\pi^2}, \tag{3.19}$$

which is indeed the one-loop gauge β-function, consistent with our earlier remark that the undeformed theory has $\mathcal{N}=2$ supersymmetry in this case.

We find from eqs. (2.19), (3.8), (3.10), (3.13), (3.15), (3.16),

$$\begin{aligned}
k_{1B}^{(1)} &= 6(k_1 + k_2 - g^2)y\overline{y}L - 3R^{(1)},\\
k_{2B}^{(1)} &= 4(k_1 + 3k_2 + 2k_3)y\overline{y}L + R^{(1)} - S^{(1)},\\
k_{3B}^{(1)} &= 2(k_1 + 5k_2 + 8k_3 + 4k_4)y\overline{y}L + S^{(1)} - T^{(1)},\\
k_{4B}^{(1)} &= 3T^{(1)},\\
K_{1B}^{(1)} &= ([6K_1 + 2K_2]y\overline{y} + 2Qg^2K_1 - g^2y\overline{y})L,\\
K_{2B}^{(1)} &= 2(12K_1 + 5K_2 + 2g^2)Qg^2L.
\end{aligned} \tag{3.20}$$

To a large extent the renormalisation of $\overline{F}_{1,2,3}$ as given in eq. (3.10) is determined by the requirement that the couplings k_{1-4}, $K_{1,2}$ are multiplicatively renormalised as described above. However we still have the freedom to choose $R^{(1)}$, $S^{(1)}$, $T^{(1)}$, which are the same for each $\overline{F}_{1,2,3B}$. Choosing $R^{(1)} = S^{(1)} = T^{(1)} = 0$ in eq. (3.10) leaves almost the minimal renormalisation of $\overline{F}_i$ possible to ensure multiplicative renormalisation; however we have included the terms with a factor Q in eq. (3.10) in order to remove g^2k_i-dependent terms in k_{1-4B} (something which is only possible thanks to the particular form of the divergences, as will become clearer later when we discuss the eliminated theory).

Writing $\beta_{k_i} = \mu\frac{d}{d\mu}k_i$ (and similarly for $K_{1,2}$) and as usual requiring that k_{iB} and $K_{1,2B}$ be independent of μ we then find that

$$\begin{aligned}
\beta_{k_1}^{(1)} &= \frac{1}{16\pi^2}[6(k_1 + k_2 - g^2)y\overline{y} - 3r],\\
\beta_{k_2}^{(1)} &= \frac{1}{16\pi^2}[4(k_1 + 3k_2 + 2k_3)y\overline{y} + r - s],\\
\beta_{k_3}^{(1)} &= \frac{1}{16\pi^2}[2(k_1 + 5k_2 + 8k_3 + 4k_4)y\overline{y} + s - t],\\
\beta_{k_4}^{(1)} &= \frac{3t}{16\pi^2},\\
\beta_{K_1}^{(1)} &= \frac{1}{16\pi^2}([6K_1 + 2K_2]y\overline{y} + 2Qg^2K_1 - g^2y\overline{y}),\\
\beta_{K_2}^{(1)} &= \frac{1}{16\pi^2}2(12K_1 + 5K_2 + 2g^2)Qg^2,
\end{aligned} \tag{3.21}$$

writing $R^{(1)} = rL$, etc. We note that these β-functions are different in form from those derived in the ungauged case in ref. [30]; of course our three-field superpotential is also somewhat different from that used in the ungauged case, and we have also had to include non-linear terms in $\overline{F}_{1B}$ (the $F_1\overline{\phi}_1^2$ terms), which removed the X_2' terms which would have spoiled renormalisability, but also contributed to k_{3B}. It seems impossible to use the freedom to choose $R^{(1)}$, $S^{(1)}$, $T^{(1)}$, in eq. (3.10) to make the two sets of β-functions agree.

We now turn to the calculation in the eliminated theory. If we eliminate F_i and $\overline{F}_i$ from the action we find

$$\begin{aligned} F_i &= \overline{G}_i, \\ \overline{F}_1 &= G_1 - \overline{y}^{-1}|C|^2\left[k_1F_2F_3 + k_2(F_2\overline{G}_3 + F_3\overline{G}_2) + k_3\overline{G}_2\overline{G}_3\right] \\ &\quad -igC^{\mu\nu}F_{\mu\nu}\overline{\phi}_1 - \frac{1}{4}g^2|C|^2K_1\overline{\phi}_1\bar{\lambda}\bar{\lambda}, \end{aligned} \tag{3.22}$$

(with corresponding expressions for $\overline{F}_2$, $\overline{F}_3$) and the action becomes

$$\begin{aligned} S = \int d^4x\Big[&-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - i\bar{\lambda}\bar{\sigma}^\mu(D_\mu\lambda) + \frac{1}{2}D^2 \\ &-igC^{\mu\nu}F_{\mu\nu}\bar{\lambda}\bar{\lambda} - i\overline{\psi}_i\bar{\sigma}^\mu(D_\mu\psi)_i - (D^\mu\overline{\phi})_i(D_\mu\phi)_i \\ &+g\sum\Big\{q_i\overline{\phi}_iD\phi_i + i\sqrt{2}gq_i(\overline{\phi}_i\lambda\psi_i - \overline{\psi}_i\bar{\lambda}\phi_i) \\ &-\gamma_iC^{\mu\nu}g\left(\sqrt{2}D_\mu\overline{\phi}_i\bar{\lambda}\bar{\sigma}_\nu\psi_i + \sqrt{2}\overline{\phi}_i\bar{\lambda}\bar{\sigma}_\nu D_\mu\psi_i\right)\Big\} + \sqrt{2}gC^{\mu\nu}D_\mu\overline{\phi}_i\bar{\lambda}\bar{\sigma}_\nu\psi_i \\ &-G_i\overline{G}_i + y(\phi_1\psi_2\psi_3 + \phi_2\psi_3\psi_1 + \phi_2\psi_3\psi_1) + \overline{y}(\overline{\phi}_1\overline{\psi}_2\overline{\psi}_3 + \overline{\phi}_2\overline{\psi}_3\overline{\psi}_1 + \overline{\phi}_2\overline{\psi}_3\overline{\psi}_1) \\ &+ig\overline{y}(1-\gamma_1-\gamma_2-\gamma_3)C^{\mu\nu}F_{\mu\nu}\overline{\phi}_1\overline{\phi}_2\overline{\phi}_3 + \lambda_1\overline{y}^{-1}|C|^2\overline{G}^3 + \lambda_2\overline{y}|C|^2\overline{\phi}_1\overline{\phi}_2\overline{\phi}_3\bar{\lambda}\bar{\lambda}\Big]. \end{aligned} \tag{3.23}$$

where

$$\begin{aligned} \lambda_1 &= k_1 + 3(k_2 + k_3) + k_4, \\ \lambda_2 &= 3K_1 + K_2. \end{aligned} \tag{3.24}$$

The renormalisation of the last three terms in eq. (3.23) now needs to be reconsidered. First let us consider the $C^{\mu\nu}F_{\mu\nu}\overline{\phi}_1\overline{\phi}_2\overline{\phi}_3$ term. Its coefficient has changed, and in particular we see, comparing eqs. (2.3), (3.23), that its finite part has changed by a factor of $-\frac{1}{2}$. Moreover the diagrams figures 3(e)-(h) which cancelled the contributions from figures 3(i)-(l) are no longer present, while these latter contributions are multiplied by $-\frac{1}{2}$. Moreover, since the eliminated theory in eq. (3.23) also contains a $\overline{G}_iG_i$ vertex which was not present in the uneliminated case, there is a new diagram depicted in figure 12, giving a divergent contribution

$$-6iy\overline{y}^2C^{\mu\nu}\int d^4xF_{\mu\nu}\overline{\phi}_1\overline{\phi}_2\overline{\phi}_3. \tag{3.25}$$

However, taking all these effects into account, it is straightforward to check that the divergences are still cancelled.

The remaining two terms need to be examined in more detail. We write the divergent contributions to these terms as

$$\Gamma^{\text{pole}}_{C\text{elim}} = -|C|^2\int d^4x[Y_1\overline{y}^{-1}\overline{G}_1\overline{G}_2\overline{G}_3 + Y_2\overline{y}\overline{\phi}_1\overline{\phi}_2\overline{\phi}_3\bar{\lambda}\bar{\lambda}], \tag{3.26}$$

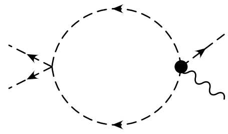

Figure 12. Additional one-loop diagram for the eliminated case.

(introducing an overall minus sign as in eq. (3.7)). Most of the relevant contributions to Y_1 can be read off from those to X_4 in table 4 with a k_4 (here replaced by λ_1). Similarly, most of the relevant contributions to Y_2 can be read off from those to X_6 in table 6 with a K_2 (here replaced by λ_2), and those to X_6 in table 8. However, in the eliminated case there are also diagrams with a $g\overline{y}C^{\mu\nu}F_{\mu\nu}\overline{\phi}_1\overline{\phi}_2\overline{\phi}_3$ vertex. Such diagrams were previously cancelled by diagrams with an internal F propagator in a similar fashion to figures 3(e)-(h) and 3(i)-(l); but of course such diagrams are no longer present in the eliminated case. Again, there are further diagrams incorporating the $\overline{G}_iG_i$ vertex which was not present in the uneliminated case. The result is that we now need to incorporate contributions from the diagrams shown in figure 13. The contributions are listed in table 9 (note that the contributions from figures 13(j), (k) cancel).

We find from the eliminated diagrams that

$$\begin{aligned} Y_1^{(1)} &= 2[12y\overline{y}\lambda_1 - (1+\alpha)g^2Q\lambda_1 - 3g^2y\overline{y}]L, \\ Y_2^{(1)} &= [6y\overline{y}\lambda_2 + (7-\alpha)Qg^2\lambda_2 + 4Qg^4 - 3g^2y\overline{y}]L, \end{aligned} \tag{3.27}$$

and so

$$\begin{aligned} \beta_{\lambda_1}^{(1)} &= \frac{1}{16\pi^2}(24\lambda_1y\overline{y} - 6g^2y\overline{y}) \\ \beta_{\lambda_2}^{(1)} &= \frac{1}{16\pi^2}(6y\overline{y}\lambda_2 + 10Qg^2\lambda_2 + 4Qg^4 - 3g^2y\overline{y}). \end{aligned} \tag{3.28}$$

An important consistency check is that

$$\begin{aligned} \lambda_{1B} &= k_{1B} + k_{4B} + 3(k_{2B} + k_{3B}), \\ \lambda_{2B} &= 3K_{1B} + K_{2B}, \end{aligned} \tag{3.29}$$

and it is easy to confirm that this is satisfied at one loop using eqs. (3.20) and (3.27). The fact that we were able to remove g^2k_i terms from $k_{iB}^{(1)}$ in the uneliminated case is now seen as a consequence of the fact that $\lambda_{1B}^{(1)}$ contains no $g^2\lambda_1$ terms.

The original deformed Wess-Zumino action of eq. (2.1) corresponded to the values $k_1 = y$, $K_1 = \frac{1}{4}g^2$, $k_{2-4} = K_2 = 0$. However, our more general Lagrangian in eq. (2.19) is invariant under $\mathcal{N} = \frac{1}{2}$ transformations whatever the values of k_{1-4}, $K_{1,2}$; and we see from eq. (3.21) that the choice $k_1 = y$, $K_1 = \frac{1}{4}g^2$, $k_{2-4} = K_2 = 0$ is not maintained by renormalisation; if we set $k_1 = y$, $K_1 = \frac{1}{4}g^2$, $k_{2-4} = K_2 = 0$ at one scale then different values are inevitably generated at other scales. In ref. [30] we asked (for the ungauged case) if there is any set of values of k_{1-4} (or at least any form for the deformed action) which *is* preserved by renormalisation and which would be in some sense natural.

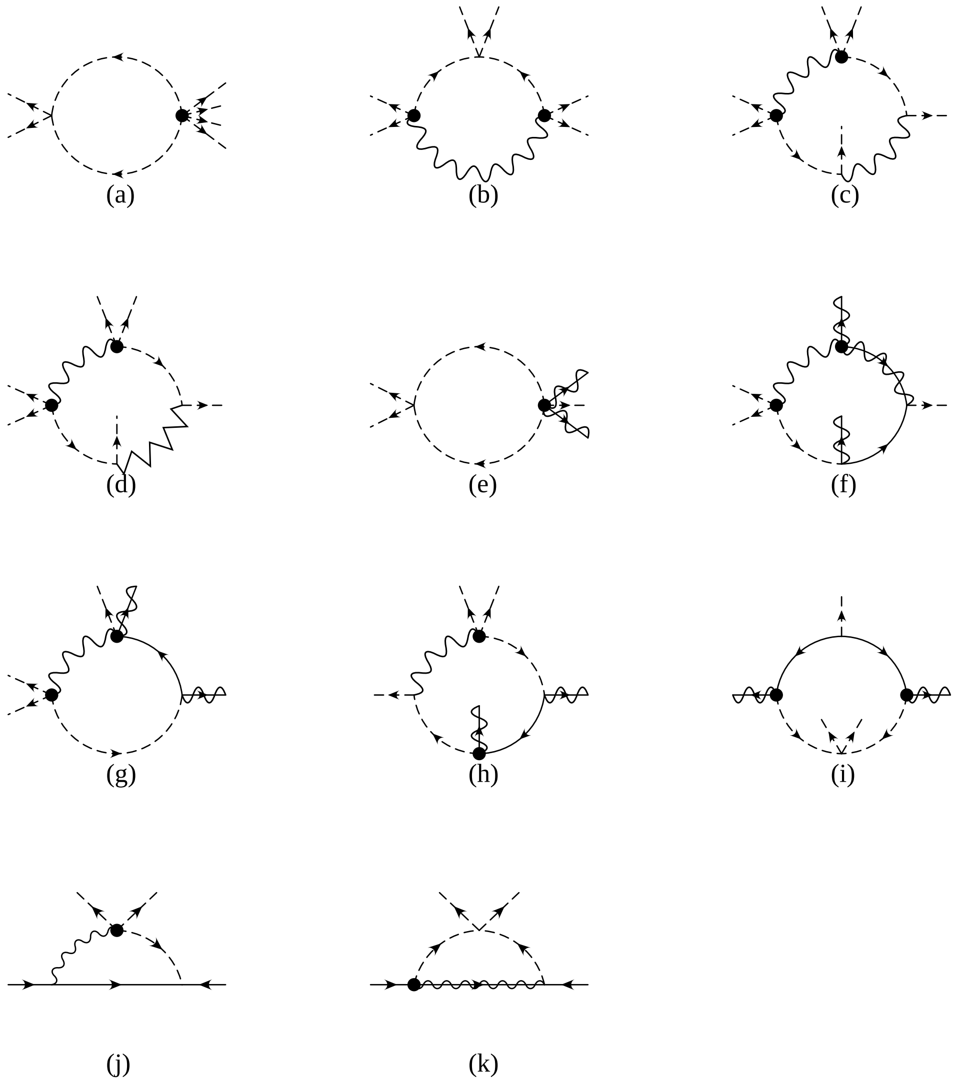

Figure 13. Further one-loop diagrams for the eliminated case.

Requiring that

$$k_i = a_i(y\overline{y})^{\rho}, \quad i = 1\dots 4,$$

where a_i, $i = 1\dots 4$ are numbers (i.e. not functions of y, $\overline{y}$, or g, and hence scale independent), entails

$$\frac{\beta_1^{(1)}}{k_1} = \frac{\beta_2^{(1)}}{k_2} = \frac{\beta_3^{(1)}}{k_3} = \frac{\beta_4^{(1)}}{k_4} = \rho\left(\frac{\beta_y^{(1)}}{y} + \frac{\beta_{\overline{y}}^{(1)}}{\overline{y}}\right). \tag{3.30}$$

If we ask the same question here we shall find that the values of k_{1-4} and ρ_i must satisfy the sole condition

$$[(24-6\rho)y\overline{y} + 4\rho Q g^2]\lambda_1 = 6g^2 y\overline{y} \tag{3.31}$$

which is the same condition we would find in the eliminated case using eq. (3.28). In the ungauged case we once again find that the particular solutions

$$k_1 = -k_2 = k_3 = -k_4, \quad \rho = 0, \tag{3.32}$$

	Y_1	Y_2
a	$24y\overline{y}\lambda_1$	
b	$-6g^2y\overline{y}$	
c	0	
d	0	
e		$6y\overline{y}\lambda_2$
f		$-8Qg^4$
g		$-2Qg^4$
h		0
i		$-3g^2y\overline{y}$

Table 9. Divergent contributions from figure 13.

and also

$$k_1 = -\frac{3}{2}k_2 = 3k_3, \quad k_4 = 0, \quad \rho = \frac{1}{3}. \tag{3.33}$$

require no non-linear renormalisation of F_i.

It is tempting to feel that there is something particularly significant about the choices in eqs. (3.32), (3.33) since they provided solutions in ref. [30] at one and two loops without the need for any further renormalisation of F_i; and in fact they also solve our current model with the β-functions in eq. (3.21), with $r = s = t = 0$, i.e. derived using the minimal renormalisation of the F_i consistent with renormalisability.

4 Conclusions

We have performed a complete one-loop analysis of the renormalisation of the simplest gauged U_1 non-anticommutative Wess-Zumino model with a superpotential. We started with the action derived from the non-anticommutative superspace theory, but then found it necessary (working with the uneliminated form of the action, without eliminating auxiliary fields) also to include all possible terms which can be generated by renormalisation with their own couplings. We showed that this leads to results compatible with those obtained in the eliminated theory. Our main results are those in eq. (3.21) (in the uneliminated case) and eq. (3.28) (in the eliminated case). This is the first complete one-loop calculation for a general non-anticommutative supersymmetric gauge theory with a superpotential; as mentioned earlier, in ref. [22] we omitted $y\overline{y}$ contributions to the renormalisation of terms in S_{kin}. The renormalisation of the theory is much simpler than in the $SU_N \times U_1$ cases considered in refs. [18, 19, 22], though once again we required a non-linear renormalisation of the gaugino λ, as parametrised by ρ_i in eq. (3.11), accompanied by a renormalisation parametrised by γ_i in eq. (2.2) (with ρ_i, γ_i as given in eq. (3.15)). These renormalisations were determined by consideration of the theory with a superpotential; however, the renormalisations contains y-independent pieces which yet would not have been required in the theory without a superpotential. It is somewhat reassuring that the y-independent part of the renormalisations for the ρ_i and γ_i is exactly as would be obtained from the U_1 part of

JHEP04(2009)028

the $SU_N \times U_1$ theory of ref. [22], despite the fact that here we have considered a trilinear, three-field superpotential and there we considered a mass term (with two fields).

Acknowledgments

RP was supported by STFC through a graduate studentship. DRTJ was visiting CERN and the Aspen Center for Physics while part of this work was carried out.

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JHEP04(2009)028